

PROBLEMES PROPOSATS

PROBLEMA N. 90

Let X be a continuous random variable such that $P(X > 0) = 1$ and X follows the same distribution as $1/X$, that is, the probability distribution function F satisfies

$$F(x) = 1 - F(1/x) \quad x > 0.$$

Prove:

- 1) The median is $M = 1$.
- 2) The mean μ , if exists, satisfies $\mu > 1$.
- 3) The variance σ^2 , if exists, satisfies $\sigma^2 \geq \mu^2 - 1$.
- 4) If μ and σ^2 exist then

$$\int_0^1 \frac{F^{-1}(t)}{F^{-1}(1-t)} dt = \sigma^2 + \mu^2.$$

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PROBLEMA N. 91

Let X be a random variable with standard logistic distribution function

$$F(x) = (1 + e^{-x})^{-1}, \quad -\infty < x < +\infty.$$

Let $G(x)$ be an absolutely continuous function. Suppose that $E(G'(X))$ and $\text{var}(G(X))$ exist. Prove the following inequality

$$3(E G'(X))^2 \leq \text{var}(G(X))$$

with equality if G is F .

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